MATHEMATICS 9th Science Group

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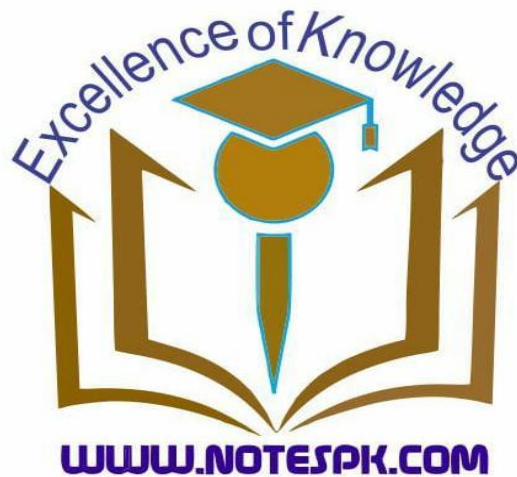
Chapter 1.

MATRICES AND DETERMINANTS

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Introduction:

The matrices and determinants are used in the field of Mathematics, Physics, Statistics, Electronics and other branches of science. The Matrices have played a very important role in this age of computer science. The idea of matrices was given by the Arthur Cayley, an English Mathematician of 19th century, who first developed, *Theory of Matrices"* "in 1858.

Matrix:

"An arrangement of different elements in the rows and columns, within square brackets is called Matrix".

e.g $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

The real numbers used in the formation of the matrix are called entries or elements of the matrix. The matrices are denoted by the capital letters $A, B, C, D, \ldots, M, N$ *etc.* of the English alphabets.

Rows and Columns of a Matrix:

In a matrix $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the horizontal way are called rows.

In a matrix $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the vertical way are called columns.

Order of a Matrix:

Order of Matrix tells us about no of rows and columns.

$\textbf{Order of a matrix} = \textbf{no. of rows} \times \textbf{no. of columns}.$

If a matrix A has m rows and n column then its order is

$$O(A) = m \times n \text{ or } m - by - n.$$

For example

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix} \text{ has order } 3-by-3 \text{ or } 3\times 3.$$

Equal matrices:

"Two matrices are said to be equal if

i. **The order of matrix A =The order of Matrix B**
ii. **Their corresponding elements are equal. Thus**

$$A = B."$$

Example:

$$A=\begin{bmatrix} 1 & 2 \\ -5 & 7 \end{bmatrix} \text{ and } B=\begin{bmatrix} 1 & 1+1 \\ -5 & 5+2 \end{bmatrix}$$

are equal matrices.

Exercise 1.1

Question.1.Find the order of the following matrices.

(i). $A=\begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$

Solution.

$\textit{Order of } A = O(A)$ =2-by-2 *or* 2×2

(ii). $B=\begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

Solution.

$\textit{Order of } B = O(B)$ =2-by-2*or* 2×2

(iii). $C=[2 \quad 4]$

Solution.

$\textit{Order of } C = O(C)$ = 1-by-2 *or* 1×2

(iv). $D=\begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

Solution.

$\textit{Order of } D = O(D)$ =3-by-1*or* 3×1

(v). $E=\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Solution.

$\textit{Order of } E = O(E)$ =3-by-2 *or* 3×2

(vi). $F=[2]$

Solution.

$\textit{Order of } F = O(F)$ =1-by-1*or* 1×1

(vii). $G=\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Solution.

$\textit{Order of } G = O(G)$ =3-by-3*or* 3×3

(viii). $H=\begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Solution.

$\textit{Order of } H = O(H)$ =2-by-3 *or* 2×3

Question.2. which of the following matrices are equal?

$A=[3]$, $B=[3 \quad 5]$

$C=[5-2]$, $D=[5 \quad 3]$

$E=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $F=\begin{bmatrix} 2 \\ 6 \end{bmatrix}$,

$G=\begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$ $H=\begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$,

$I=[3 \quad 3+2]$, $J=\begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Solution.

From above matrices

$$A = C$$
$$E = H = J$$
$$F = G$$

Question.3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution.
Given
$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$
By the definition of equal matrices, we have
$$a+c=0 \to (i)\ , a+2b=-7 \to (ii)\ ,$$
$$c-1=3 \to (iii)\ , 4d-6=2d \to (iv)$$
From (iii), we have
$$c=3+1=4$$
From (iv), we have
$$4d-6=2d$$
$$4d-2d=6$$
$$2d=6$$
$$d=\frac{6}{2}$$
$$d=3$$
Using value of $c=4\ in\ (i), we\ have$
$$a+4=0$$
$$a=-4$$
Using value of $a=-4\ in\ (ii), we\ have$
$$-4+2b=-7$$
$$2b=-7+4$$
$$2b=-3$$
$$b=-\frac{3}{2}$$
Hence $a=-4$, $b=-\frac{3}{2}$, $c=4\ and\ d=3.$

Types of Matrices:

Row matrix:

"A matrix having single row is called Row Matrix."

Example:

$M=[1\ \ 2\ \ 3]\ is\ a\ row\ matrix\ of\ order\ 1-by-3.$

Column matrix:

A matrix having single column is called column Matrix.

Example:

$$M=\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}\ is\ a\ column\ matrix\ of\ order\ 3-by-1.$$

Rectangular matrix:

A matrix in which number of rows is not equal to number of columns is called rectangular Matrix.

Example:

$$\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix} \text{ are rectangular matrices.}$$

Square matrix:

"A matrix in which number of rows is equal to the number of columns then matrix is called square matrix."

Example:

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix}\ has\ order\ 3-by-3.$$

Null or Zero Matrix:

"A matrix whose each element is zero, is called a null or zero matrix. It is denoted by O."

Examples:

$$[0]\ ,\ [0\ \ 0]\ ,\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ are null matrices.}$$

Transpose of a Matrix:

"A matrix obtained by changing the rows into columns or columns into rows of a matrix is called transpose of that matrix. If A is a matrix, then its transpose matrix is denoted by A^t."

Example:

$$\text{If } A=\begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix}\ then\ A^t=\begin{bmatrix} 1 & 9 & 4 \\ 2 & 7 & 6 \\ 3 & 6 & 8 \end{bmatrix}$$

$$\text{If } B=\begin{bmatrix} 1 & 3 & 2 \\ 1 & 9 & 4 \end{bmatrix}\ then\ B^t=\begin{bmatrix} 1 & 1 \\ 3 & 9 \\ 2 & 4 \end{bmatrix}$$

If a matrix B is of order 2-by-3 then order its transpose matrix B^t is 3-by-2.

Negative of a Matrix:

"Let A be a matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A."

Example:

$$\text{If } A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\ , then\ -A=\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}.$$

Symmetric matrix:

"Let A be the square matrix, if $A^t=A$ then A is called symmetric matrix."

Example:

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}\ is\ a\ square\ matrix\ then$$
$$A^t=\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}=A.$$
$$Thus\ A\ is\ symmteric\ matrix.$$

Skew-symmetric matrix:

"Let A be the square matrix, if $A^t=-A$ then A is called skew symmetric matrix."

Example:

$$A=\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}\ is\ a\ square\ matrix\ then$$
$$A^t=\begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}=-\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$$
$$=-A.$$
$$Thus\ A\ is\ a\ skew-symmteric\ matrix.$$

Diagonal matrix:

"A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not

zero and non-diagonal entries are zero."

Example:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$are\ called\ diagonal\ matrices.$

Scalar Matrix:

"A diagonal matrix having same elements in principle diagonal except 1 or 0 is called scalar matrix."

Example:

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \text{ are Scalar matrices.}$$

Unit Matrix or Identity Matrix:

A diagonal matrix is called identity matrix if all diagonal entries are 1. It is denoted by I.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ are identity matrices.}$$

Exercise 1.2

Question.1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = [2 \quad 3 \quad 4], \quad \boldsymbol{C} = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{E} = [0], \quad \boldsymbol{F} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

Solution.

Identity Matrices: D

Row Matrices: B and E.

Column Matrices: C, E and F.

Null Matrices: A and E.

Question.2. From the following matrices, identify (a) Square matrices, (b) Rectangular matrices, (c) Row matrices, (d) Column matrices, (e) Identity Matrices, (f) Null matrices.

(i). $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ $(ii).\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ $(iii).\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$ $(iv).\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(v)\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(vi). $[3 \quad 10 \quad -1]$ **(vii).** $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $(viii).\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $(ix).\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution.

(a). Square Matrices: (iii). (iv). (viii).

(b). Rectangular Matrices: (i). (ii). (v).

(c). Row Matrices: (vi).

(d). (ii). (vii).

(e). (iv).

(f). (ix).

Question.3. From the following matrices, identify Diagonal matrices, Scalar matrices and Unit (identity) matrices.

$$\boldsymbol{A} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{E} = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$$

Solution.

Diagonal Matrices: A, B, C, D, E.

Scalar Matrices: A, C, E.

Unit Matrices: C.

Question.4. Find the negative of matrices A, B, C, D and E when:

(i). $\boldsymbol{A} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Solution.

$$-A = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(ii). $B = [5 \quad 1 \quad -6]$

Solution.

$$-B = [-5 \quad -1 \quad 6]$$

(iii). $C = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

Solution.

$$-C = \begin{bmatrix} -2 & -3 \\ 0 & -5 \end{bmatrix}$$

(iv). $D = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

Solution.

$$-D = \begin{bmatrix} -2 & -3 \\ 4 & -5 \end{bmatrix}$$

(v). $E = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution.

$$-E = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

Question.5. Find the transpose of the following matrices:

(i). $\boldsymbol{A} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$

Solution.

$$\boldsymbol{A^t} = [0 \quad 1 \quad -1]$$

(ii). $\boldsymbol{C} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$

Solution.

$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(iii). $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

Solution.

$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(iv). $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$

Solution.

$$E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

(v). $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution.

$$F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Question.6. Verify that if $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$, *then*

(i). $(A^t)^t = A$

Solution.

Given

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$(A^t)^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = A$$

$$(A^t)^t = A$$

Hence Proved.

(ii). $(B^t)^t = B$

Solution.

Given

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$

$$(B^t)^t = B$$

Hence Proved

Addition of matrices:

"Let A and B be any two matrices of same order then A and B are comfortable for addition." Addition of A and B, Written as $A + B$ is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B."

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ *are comfortable for addition.*

$$A + B = \begin{bmatrix} 2-2 & 3+3 & 0+4 \\ 1+1 & 0+2 & 6+3 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 4 \\ 2 & 2 & 9 \end{bmatrix}$$

Subtraction of matrices:

Let A and B be any two matrices of same order then A and B are comfortable for Subtraction. Subtraction of A and B, Written as $A - B$ is obtained by subtracting the entries of the matrix A to the corresponding entries of the matrix B.

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ *are comfortable for* Subtraction.

$$A - B = \begin{bmatrix} 2+2 & 3-3 & 0-4 \\ 1-1 & 0-2 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -4 \\ 0 & -2 & 3 \end{bmatrix}$$

Multiplication of a Matrix by a Real Number:

Let A be any matrix and the real number k be a scalar. Then the scalar multiplication of matrix A with k is obtained by multiplying each entry of matrix A with k. It is denoted by kA.

Example:

$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix}$ $then\ kA = \begin{bmatrix} 2k & 3k & 0 \\ 1k & 0 & 6k \end{bmatrix}$

Commutative Law for Addition.

If A and B are two matrices of the same order, Then $A + B = B + A$ *is* called commutative law under addition.

$$A + B = B + A$$

Associative Law for Addition:

If A, B and C are three matrices of the same order, Then $(A + B) + C = A + (B + C)$ *is* Called Associative law under addition.

$$(A + B) + C = A + (B + C)$$

Additive Identity of a Matrix:

If A and B are two matrices of same order and $A + B = A = B + A$

Then matrix B is called additive identity of matrix A. For any matrix A and zero matrix of same order, O is called additive identity of A as

$$A + O = A = O + A.$$

Additive Inverse of a Matrix:

If A and B are two matrices of same order and $A + B = O = B + A$

Then matrix B is called additive inverse of matrix A. "Additive inverse of any matrix A is obtained by changing to negative of the symbols (entries) of each non zero entry of A."

Exercise 1.3

Question.1. which of the following matrices are comfortable for addition?

$$A=\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix},\quad B=\begin{bmatrix}3\\1\end{bmatrix},\quad C=\begin{bmatrix}1 & 0\\ 2 & -1\\ 1 & -2\end{bmatrix}$$

$$D=\begin{bmatrix}2+1\\3\end{bmatrix},\quad E=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix},$$

$$F=\begin{bmatrix}3 & 2\\ 1+1 & -4\\ 3+2 & 2+1\end{bmatrix}$$

Solution.

Since order of A and E are same so they are comfortable for addition.

Also order of B and D are same so they are comfortable for addition.

Also order of C and F are same so they are comfortable for addition.

Question.2. Find the additive inverse of the following matrices:

(i). $A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}$

Solution.

$$\text{Additive inverse of } A=-A=\begin{bmatrix}2 & 4\\ -2 & 1\end{bmatrix}$$

(ii). $B=\begin{bmatrix}1 & 0 & -1\\ 2 & -1 & 3\\ 3 & -2 & 1\end{bmatrix}$

Solution.

$$\text{Additive inverse of } B=-B=\begin{bmatrix}-1 & 0 & 1\\ -2 & 1 & -3\\ -3 & 2 & -1\end{bmatrix}$$

(iii). $C=\begin{bmatrix}4\\-2\end{bmatrix}$

Solution.

$$\text{Additive inverse of } C=-C=\begin{bmatrix}-4\\2\end{bmatrix}$$

(iv). $D=\begin{bmatrix}1 & 0\\ -3 & -2\\ 2 & 1\end{bmatrix}$

Solution.

$$\text{Additive inverse of } D=-D=\begin{bmatrix}-1 & 0\\ 3 & 2\\ -2 & -1\end{bmatrix}$$

(v). $E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Solution.

$$\text{Additive inverse of } E=-E=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

(vi). $F=\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{2}\end{bmatrix}$

Solution.

$$\text{Additive inverse of } F=-F=\begin{bmatrix}-\sqrt{3} & -1\\ 1 & -\sqrt{2}\end{bmatrix}$$

Question.3.If $A=\begin{bmatrix}-1 & 2\\ -2 & 1\end{bmatrix}, B=\begin{bmatrix}1\\-1\end{bmatrix}, C=[1\quad -1\quad 2], D=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$

Then find,

(i). $A+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

Solution.

$$A+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}=\begin{bmatrix}-1 & 2\\ -2 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 2+1\\ -2+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 3\\ -1 & 2\end{bmatrix}$$

Answer.

(ii). $B+\begin{bmatrix}-2\\3\end{bmatrix}$

Solution.

$$B+\begin{bmatrix}-2\\3\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix}+\begin{bmatrix}-2\\3\end{bmatrix}$$

$$=\begin{bmatrix}1-2\\-1+3\end{bmatrix}$$

$$=\begin{bmatrix}-1\\2\end{bmatrix}$$

Answer.

(iii). $C+[-2\quad 1\quad 3]$

Solution.

$$C+[-2\quad 1\quad 3]=[1\quad -1\quad 2]+[-2\quad 1\quad 3]$$

$$=[1-2\quad -1+1\quad 2+3]$$

$$=[-1\quad 0\quad 5]$$

Answer.

(iv). $D+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}$

Solution.

$$D+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & 0\\ 2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 2+1 & 3+0\\ -1+2 & 0+0 & 2+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 3 & 3\\ 1 & 0 & 3\end{bmatrix}$$

Answer.

(v). $2A$

Solution.

$$2A=2\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2 & 4\\ 4 & 2\end{bmatrix}$$

Answer.

(vi). $(-1)B$

Solution.

$$(-1)B=(-1)\begin{bmatrix}1\\-1\end{bmatrix}$$

$$=\begin{bmatrix}-1\\1\end{bmatrix}$$

Answer.

(vii). $(-2)C$

Solution.

$$(-2)C=(-2)[1\quad -1\quad 2]$$

$$=[-1\quad 2\quad -4]$$

Answer.

(viii). $3D$

Solution.

$$3D=3\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3 & 6 & 9\\ -3 & 0 & 6\end{bmatrix}$$

Answer.

(ix). $3C$

Solution.

$$3C=3[1\quad -1\quad 2]$$

$$= [3 \quad -6 \quad 6]$$

Answer.

Question.4. perform the indicated operations and simplify the following

(i). $\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$

Solution.

$$\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$
$$= \left(\begin{bmatrix}1+0 & 0+2\\0+3 & 1+0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$
$$= \begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$$
$$= \begin{bmatrix}1+1 & 2+1\\3+0 & 1+1\end{bmatrix}$$
$$= \begin{bmatrix}2 & 3\\3 & 2\end{bmatrix}$$

Answer.

(ii). $\begin{bmatrix}\mathbf{1} & \mathbf{0}\\\mathbf{0} & \mathbf{1}\end{bmatrix}+\left(\begin{bmatrix}\mathbf{0} & \mathbf{2}\\\mathbf{3} & \mathbf{0}\end{bmatrix}-\begin{bmatrix}\mathbf{1} & \mathbf{1}\\\mathbf{1} & \mathbf{0}\end{bmatrix}\right)$

Solution.

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\left(\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\right)$$
$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\left(\begin{bmatrix}0-1 & 2-1\\3-1 & 0-1\end{bmatrix}\right)$$
$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\2 & -1\end{bmatrix}$$
$$= \begin{bmatrix}1-1 & 0+1\\0+2 & 1-1\end{bmatrix}$$
$$= \begin{bmatrix}0 & 1\\2 & 0\end{bmatrix}$$

Answer.

(iii). $[\mathbf{2} \quad \mathbf{3} \quad \mathbf{1}]+([\mathbf{1} \quad \mathbf{0} \quad \mathbf{2}]-[\mathbf{2} \quad \mathbf{2} \quad \mathbf{2}])$

Solution.

$$[2 \quad 3 \quad 1]+([1 \quad 0 \quad 2]-[2 \quad 2 \quad 2])$$
$$= [2 \quad 3 \quad 1] + ([1-2 \quad 0-2 \quad 2-2])$$
$$= [2 \quad 3 \quad 1]+[-1 \quad -2 \quad 0]$$
$$= [2-1 \quad 3-2 \quad 1+0]$$
$$= [1 \quad 1 \quad 1]$$

Answer.

(iv). $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\\mathbf{-1} & \mathbf{-1} & \mathbf{-1}\\\mathbf{0} & \mathbf{1} & \mathbf{2}\end{bmatrix}+\begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\\\mathbf{2} & \mathbf{2} & \mathbf{2}\\\mathbf{3} & \mathbf{3} & \mathbf{3}\end{bmatrix}$

Solution.

$$\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$$
$$= \begin{bmatrix}1+1 & 2+1 & 3+1\\-1+2 & -1+2 & -1+2\\0+3 & 1+3 & 2+3\end{bmatrix}$$
$$= \begin{bmatrix}2 & 3 & 4\\1 & 1 & 1\\3 & 4 & 5\end{bmatrix}$$

Answer.

(vi). $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\\mathbf{2} & \mathbf{3} & \mathbf{1}\\\mathbf{3} & \mathbf{1} & \mathbf{2}\end{bmatrix}+\begin{bmatrix}\mathbf{1} & \mathbf{0} & \mathbf{-2}\\\mathbf{-2} & \mathbf{-1} & \mathbf{0}\\\mathbf{0} & \mathbf{2} & \mathbf{-1}\end{bmatrix}$

Solution.

$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$$
$$= \begin{bmatrix}1+1 & 2+0 & 3-2\\2-1 & 3-1 & 1+0\\3+0 & 1+2 & 2-1\end{bmatrix}$$
$$= \begin{bmatrix}2 & 2 & 1\\1 & 2 & 1\\3 & 3 & 1\end{bmatrix}$$

Answer.

(vi). $\left(\begin{bmatrix}\mathbf{1} & \mathbf{2}\\\mathbf{0} & \mathbf{1}\end{bmatrix}+\begin{bmatrix}\mathbf{1} & \mathbf{2}\\\mathbf{0} & \mathbf{1}\end{bmatrix}\right)+\begin{bmatrix}\mathbf{1} & \mathbf{1}\\\mathbf{1} & \mathbf{1}\end{bmatrix}$

Solution.

$$\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$
$$= \left(\begin{bmatrix}1+1 & 2+2\\0+0 & 1+1\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$
$$= \begin{bmatrix}2 & 4\\0 & 2\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$
$$= \begin{bmatrix}2+1 & 4+1\\0+1 & 2+1\end{bmatrix}$$
$$= \begin{bmatrix}3 & 5\\1 & 3\end{bmatrix}$$

Answer.

Question5.For the matrices $\boldsymbol{A} = \begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\\mathbf{2} & \mathbf{3} & \mathbf{1}\\\mathbf{1} & \mathbf{-1} & \mathbf{0}\end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix}\mathbf{1} & \mathbf{-1} & \mathbf{1}\\\mathbf{2} & \mathbf{-2} & \mathbf{2}\\\mathbf{3} & \mathbf{1} & \mathbf{3}\end{bmatrix}$,

$\boldsymbol{C} = \begin{bmatrix}\mathbf{-1} & \mathbf{0} & \mathbf{0}\\\mathbf{0} & \mathbf{-2} & \mathbf{3}\\\mathbf{1} & \mathbf{1} & \mathbf{2}\end{bmatrix}$, **Verify the following rules:**

(i). $\boldsymbol{A + C = C + A}$

Solution.

$$L.H.S = A + C$$
$$L.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & -0 & 2\end{bmatrix}$$
$$R.H.S = C + A$$
$$R.H.S = \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2+0\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & -0 & 2\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(ii). $\boldsymbol{A + B = B + A}$

Solution.

$$L.H.S = A + B$$
$$L.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$
$R.H.S=B+A$
$$R.H.S=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+1 & 1-1 & 3+0\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$
Hence Proved $L.H.S=R.H.S.$

(iii). $\boldsymbol{B+C=C+B}$

Solution.

$L.H.S=B+C$
$$L.H.S=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}1-1 & -1+0 & 1+0\\2+0 & -2-2 & 2+3\\3+1 & 1+1 & 3+2\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$$
$R.H.S=C+B$
$$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}-1+1 & 0-1 & 0+1\\0+2 & -2-2 & 3+2\\1+3 & 1+1 & 2+3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$$
Hence Proved $L.H.S=R.H.S.$

(iv). $\boldsymbol{A+(B+A)=2A+B}$

Solution.

$L.H.S=A+(B+A)$
$$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}\right)$$
$$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1+1 & -1+2 & 1+3\\2+2 & -2+3 & 2+1\\3+1 & 1-1 & 3+0\end{bmatrix}\right)$$
$$L.H.S=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$
$R.H.S=2A+B$
$$R.H.S=2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}2+1 & 4-1 & 6+1\\4+2 & 6-2 & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$
Hence Proved $L.H.S=R.H.S.$

(v). $\boldsymbol{(C-B)+A=C+(A-B)}$

Solution.

$L.H.S=(C-B)+A$
$$L.H.S=\left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S=\left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}-2+1 & 1+2 & -1+3\\-2+2 & 0+3 & 1+1\\-2+1 & 0-1 & -1+0\end{bmatrix}$$
$$L.H.S=\begin{bmatrix}-1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}$$
$R.H.S=C+(A-B)$
$$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}-\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$
$$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\left(\begin{bmatrix}1-1 & 2+1 & 3-1\\2-2 & 3+2 & 1-2\\1-3 & -1-1 & 0-3\end{bmatrix}\right)$$
$$R.H.S=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 3 & 2\\0 & 5 & -1\\-2 & -2 & -3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}-1+0 & 0+3 & 0+2\\0+0 & -2+5 & 3-1\\1-2 & 1-2 & 2-3\end{bmatrix}$$
$$R.H.S=\begin{bmatrix}-1 & 3 & 2\\0 & 3 & 2\\-1 & -1 & -1\end{bmatrix}$$
Hence Proved $L.H.S=R.H.S.$

(vi). $\boldsymbol{2A+B=A+(A+B)}$

Solution.

$L.H.S=2A+B$

$$L.H.S = 2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2+1 & 4-1 & 6+1\\4+2 & 6-2 & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

$R.H.S = A + (A + B)$

$$L.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1!+3 & -1+1 & 0+3\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(vii). $(\boldsymbol{C} - \boldsymbol{B}) - \boldsymbol{A} = (\boldsymbol{C} - \boldsymbol{A}) - \boldsymbol{B}$

Solution.

$L.H.S = (C - B) - A$

$$L.H.S = \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0+1 & -1-0\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

$R.H.S = (C - A) - B$

$$R.H.S = \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0+1 & -1-1\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(viii). $(\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C} = \boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C})$

Solution.

$L.H.S = (A + B) + C$

$$L.H.S = \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}2-1 & 1+0 & 4+0\\4+0 & 1-2 & 3+6\\4+1 & 0+1 & 3+2\end{bmatrix}$$

$$L.H.S = \begin{bmatrix}1 & 1 & 4\\4 & -1 & 9\\5 & 1 & 5\end{bmatrix}$$

$R.H.S = A + (B + C)$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1-1 & -1+0 & 1+0\\2+0 & -2-2 & 2+3\\3+1 & 1+1 & 3+2\end{bmatrix}\right)$$

$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}0 & -1 & 1\\2 & -4 & 5\\4 & 2 & 5\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1+0 & 2-1 & 3+1\\2+2 & 3-4 & 1+5\\1+4 & -1+2 & 0+5\end{bmatrix}$$

$$R.H.S = \begin{bmatrix}1 & 1 & 4\\4 & -1 & 9\\5 & 1 & 5\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

(ix). $\boldsymbol{A} + (\boldsymbol{B} - \boldsymbol{C}) = (\boldsymbol{A} - \boldsymbol{C}) + \boldsymbol{B}$

Solution.

$$L.H.S = A + (B - C)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \left(\begin{bmatrix} 1+1 & -1-0 & 1-0 \\ 2-0 & -2+2 & 2-3 \\ 3-1 & 1-1 & 3-2 \end{bmatrix} \right)$$

$$L.H.S = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 1+2 & 2-1 & 3+1 \\ 2+2 & 3+0 & 1-1 \\ 1+2 & -1+0 & 0+1 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

$$R.H.S = (A - C) + B$$

$$R.H.S = \left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \left(\begin{bmatrix} 1+1 & 2-0 & 3-0 \\ 2-0 & 3+2 & 1-3 \\ 1-1 & -1-1 & 0-2 \end{bmatrix} \right) + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -2 \\ 0 & -2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 2+1 & 2-1 & 3+1 \\ 2+2 & 5-2 & -2+2 \\ 0+3 & -2+1 & -2+3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 3 & 1 & 4 \\ 4 & 3 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

Hence Proved. $\boldsymbol{L.H.S = R.H.S}$.

(x). $\boldsymbol{2A + 2B = 2(A + B)}$

Solution.

$$L.H.S = 2A + 2B$$

$$L.H.S = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix}$$

$$L.H.S = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 4 & 6 \\ 7 & 0 & 6 \end{bmatrix}$$

$$R.H.S = 2(A + B)$$

$$L.H.S = 2\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} \right)$$

$$R.H.S = 2\left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1!+3 & -1+1 & 0+3 \end{bmatrix} \right)$$

$$R.H.S = 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 4 & 6 \\ 7 & 0 & 6 \end{bmatrix}$$

Hence Proved $L.H.S = R.H.S$.

Question.6. If $\boldsymbol{A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}}$ ***and*** $\boldsymbol{B =}$ $\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$ *, find*

(i). $3A - 2B$

Solution.

$$3A - 2B = 3\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

Answer.

(ii). $\boldsymbol{2A^t - 3B^t}$

Solution.

$$2A^t - 3B^t = 2\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}^t - 3\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}^t$$

$$= 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} - 3\begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 2-0 & 6+9 \\ -4-21 & 8-24 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$$

Answer.

Question.7. If $\boldsymbol{2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} =}$ $\boldsymbol{\begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}}$***, find***

Solution.

Given that

$$2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+3 & 8+3b \\ -6+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$$

By the definition of the equal matrix, we have

$$8 + 3b = 10\ , \quad 2a - 12 = 1$$

$$3b = 10 - 8\ , \quad 2a = 1 + 12$$

$$b = \frac{2}{3}\ , \quad a = \frac{13}{2}$$

Hence $a = \frac{13}{2}$ *and* $b = \frac{2}{3}$.

Question.8. If $\boldsymbol{A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B =}$ $\boldsymbol{\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}}$ ***, then verify that***

(i). $\boldsymbol{(A + B)^t = A^t + B^t}$

Solution.

$$L.H.S = (A + B)^t$$

$$L.H.S = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \right)^t$$

$$L.H.S = \left(\begin{bmatrix}1+1 & 2+1\\0+2 & 1+0\end{bmatrix}\right)^t$$
$$L.H.S = \begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}^t$$
$$L.H.S = \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$
$$R.H.S = A^t + B^t$$
$$R.H.S = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$R.H.S = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}1+1 & 0+2\\2+1 & 1+0\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$$

Hence Proved. $L.H.S = R.H.S.$

(ii). $(\boldsymbol{A-B})^t = \boldsymbol{A^t - B^t}$

Solution.

$$L.H.S = (A-B)^t$$
$$L.H.S = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t$$
$$L.H.S = \left(\begin{bmatrix}1-1 & 2-1\\0-2 & 1-0\end{bmatrix}\right)^t$$
$$L.H.S = \begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}^t$$
$$L.H.S = \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$
$$R.H.S = A^t - B^t$$
$$R.H.S = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$R.H.S = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}1-1 & 0-2\\2-1 & 1-0\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$$

Hence Proved. $L.H.S = R.H.S.$

(iii). $\boldsymbol{A + A^t}$ ***is symmetric.***

Solution.

$$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$
$$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$
$$A + A^t = \begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix}$$
$$A + A^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} --- (i)$$

Now

$$(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t$$
$$(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$$

Using equation (i), we have

$$(A + A^t)^t = A + A^t$$

Hence $A + A^t$ *is symmetric.*

(iv). $\boldsymbol{A - A^t}$ ***is Skew − symmetric.***

Solution.

$$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t$$
$$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$
$$A - A^t = \begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix}$$
$$A - A^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix} --- (i)$$

Now

$$(A - A^t)^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t$$
$$(A - A^t)^t = \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$$
$$(A - A^t)^t = -\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

Using equation (i), we have

$$(A - A^t)^t = -(A - A^t)$$

Hence $A - A^t$ *is Skew − symmetric.*

(iii). $\boldsymbol{B + B^t}$ ***is symmetric.***

Solution.

$$B + B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$B + B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$B + B^t = \begin{bmatrix}1+1 & 1+2\\2+1 & 0+0\end{bmatrix}$$
$$B + B^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix} --- (i)$$

Now

$$(B + B^t)^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}^t$$
$$(B + B^t)^t = \begin{bmatrix}2 & 3\\3 & 0\end{bmatrix}$$

Using equation (i), we have

$$(B + B^t)^t = B + B^t$$

Hence $B + B^t$ *is symmetric.*

(iii). $\boldsymbol{B - B^t}$ ***is Skew − symmetric.***

Solution.

$$B - B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t$$
$$B - B^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$$
$$B - B^t = \begin{bmatrix}1-1 & 1-2\\2-1 & 0-0\end{bmatrix}$$
$$B - B^t = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} --- (i)$$

Now

$$(B - B^t)^t = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}^t$$
$$(B - B^t)^t = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$
$$(B - B^t)^t = -\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$$

Using equation (i), we have

$$(B - B^t)^t = -(B - B^t)$$

Multiplication of Matrices:
Two matrices A and B are conformable for multiplication if
No of col of A =No. Of Rows of B

Exercise1.4

Q#1) Which of the following product matrices is conformable for multiplication?.

(i). $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(ii). $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(iii). $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

Sol:
Not conformable for multiplication because
No of col of 1st Matrix= 1 ≠ 2 =No. Of Rows of 2nd Matrix

(iv). $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

(v). $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Sol:
Conformable for multiplication because
No of col of 1st Matrix= 3 =No. Of Rows of 2nd Matrix

Q#2) If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$
Find (i). AB
(ii). BA (if possible)

(i). AB
Sol:

$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} (3)(6) + (0)(5) \\ (-1)(6) + (2)(5) \end{bmatrix}$

$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$

$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii). BA (if possible)
Sol:

$BA = \begin{bmatrix} 6 \\ 5 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

Since
No of col of $A = 1 \neq 2$ =No. Of Rows of B
Multiplication is not possible.

Q#3) Find the following products.

(i). $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$= [(1)(4) + (2)(0)]$
$= [4 + 0]$
$= [4]$

(ii). $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$
$= [(1)(5) + (2)(-4)]$
$= [5 - 8]$
$= [-3]$

(iii). $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$= [(-3)(4) + (0)(0)]$
$= [-12 + 0]$
$= [-12]$

(iv). $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Sol: $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
$= [(6)(4) + (0)(0)]$
$= [24 + 0]$
$= [24]$

(v). $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Sol: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$= \begin{bmatrix} (1)(4) + (2)(0) & (1)(5) + (2)(-4) \\ (-3)(4) + (0)(0) & (-3)(5) + (0)(-4) \\ (6)(4) + (-1)(0) & (6)(5) + (-1)(-4) \end{bmatrix}$

$= \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 + 0 \\ 24 + 0 & 30 + 4 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$

Q#4) Multiply the following matrices.

(a). $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Sol: $\begin{bmatrix}2 & 3\\1 & 1\\0 & -2\end{bmatrix}\begin{bmatrix}2 & -1\\3 & 0\end{bmatrix}$

$=\begin{bmatrix}(2)(2)+(3)(3) & (2)(-1)+(3)(0)\\(1)(2)+(1)(3) & (1)(-1)+(1)(0)\\(0)(2)+(-2)(3) & (0)(-1)+(-2)(0)\end{bmatrix}$

$=\begin{bmatrix}4+9 & -2+0\\2+3 & -1+0\\0-6 & 0+0\end{bmatrix}$

$=\begin{bmatrix}13 & -2\\5 & -1\\-6 & 0\end{bmatrix}$

(b). $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

Sol: $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}$

$=\begin{bmatrix}(1)(1)+(2)(3)+(3)(-1) & (1)(2)+(2)(4)+(3)(1)\\(4)(1)+(5)(3)+(6)(-1) & (4)(2)+(5)(4)+(6)(1)\end{bmatrix}$

$=\begin{bmatrix}1+6-3 & 2+8+3\\4+15-6 & 8+20+6\end{bmatrix}$

$=\begin{bmatrix}4 & 13\\13 & 34\end{bmatrix}$

(c). $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

Sol: $\begin{bmatrix}1 & 2\\3 & 4\\-1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}$

$=\begin{bmatrix}(1)(1)+(2)(4) & (1)(2)+(2)(5) & (1)(3)+(2)(6)\\(3)(1)+(4)(4) & (3)(2)+(4)(5) & (3)(3)+(4)(6)\\(-1)(1)+(1)(4) & (-1)(2)+(1)(5) & (-1)(3)+(1)(6)\end{bmatrix}$

$=\begin{bmatrix}1+8 & 2+10 & 3+12\\3+16 & 6+20 & 9+24\\-1+4 & -2+5 & -3+6\end{bmatrix}$

$=\begin{bmatrix}9 & 12 & 15\\19 & 26 & 33\\3 & 3 & 3\end{bmatrix}$

(d). $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

Sol: $\begin{bmatrix}8 & 5\\6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\-4 & 4\end{bmatrix}$

$=\begin{bmatrix}(8)(2)+(5)(-4) & (8)\left(-\frac{5}{2}\right)+(5)(4)\\(6)(2)+(4)(-4) & (6)\left(-\frac{5}{2}\right)+(4)(4)\end{bmatrix}$

$=\begin{bmatrix}16-20 & -20+20\\12-16 & -15+16\end{bmatrix}$

$=\begin{bmatrix}-4 & 0\\-4 & 1\end{bmatrix}$

(e). $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Sol: $\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$=\begin{bmatrix}(-1)(0)+(2)(0) & (-1)(0)+(2)(0)\\(1)(0)+(1)(0) & (1)(0)+(1)(0)\end{bmatrix}$

$=\begin{bmatrix}0+0 & 0+0\\0+0 & 0+0\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

Q#5) Let $A=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$, **Verify that**

(i). $AB=BA$

Sol: $L.H.S=AB$

$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}$

$=\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\(2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}$

$=\begin{bmatrix}-1-9 & -2-15\\2+0 & 4+0\end{bmatrix}$

$=\begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}\rightarrow(1)$

$: R.H.S=BA$

$=\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}$

$=\begin{bmatrix}(1)(-1)+(2)(2) & (1)(3)+(2)(0)\\(-3)(-1)+(-5)(2) & (-3)(3)+(-5)(0)\end{bmatrix}$

$=\begin{bmatrix}-1+4 & 3+0\\3-10 & -6+0\end{bmatrix}$

$=\begin{bmatrix}3 & 3\\-7 & -6\end{bmatrix}\rightarrow(2)$

From (1) and (2), we have

$AB\neq BA$

(ii). $A(BC)=(AB)C$

Sol:

$L.H.S=A(BC)$

$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}\right)$

$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\left(\begin{bmatrix}(1)(2)+(2)(1) & (1)(1)+(2)(3)\\(-3)(2)+(-5)(1) & (-3)(1)+(-5)(3)\end{bmatrix}\right)$

$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}2+2 & 1+6\\-6-5 & -3-15\end{bmatrix}$

$=\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\-11 & -18\end{bmatrix}$

$=\begin{bmatrix}(-1)(4)+(3)(-11) & (-1)(7)+(3)(-18)\\(2)(4)+(0)(-11) & (2)(7)+(0)(-18)\end{bmatrix}$

$=\begin{bmatrix}-4-33 & -7-54\\8+0 & 14+0\end{bmatrix}$

$=\begin{bmatrix}-37 & -61\\8 & 14\end{bmatrix}\rightarrow(1)$

$R.H.S=(AB)C$

$=\left(\begin{bmatrix}-1 & 3\\2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\-3 & -5\end{bmatrix}\right)\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$=\left(\begin{bmatrix}(-1)(1)+(3)(-3) & (-1)(2)+(3)(-5)\\(2)(1)+(0)(-3) & (2)(2)+(0)(-5)\end{bmatrix}\right)\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$=\begin{bmatrix}-1-9 & -2-15\\2+0 & 4+0\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$=\begin{bmatrix}-10 & -17\\2 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$

$=\begin{bmatrix}(-10)(2)+(-17)(1) & (-10)(1)+(-17)(3)\\(2)(2)+(4)(1) & (2)(1)+(4)(3)\end{bmatrix}$

$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$

$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \ldots(2)$

From (1) and (2), we have

$A(BC) = (AB)C$

(iii). $\boldsymbol{A(B+C) = AB + AC}$

Sol:

$L.H.S = A(BC)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} (-1)(3)+(3)(-2) & (-1)(3)+(3)(-2) \\ (2)(3)+(0)(-2) & (2)(3)+(0)(-2) \end{bmatrix}$

$= \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \rightarrow(1)$

$R.H.S = AB + BC$

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) + \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$+ \begin{bmatrix} (-1)(2)+(3)(1) & (-1)(1)+(3)(3) \\ (2)(2)+(0)(1) & (2)(1)+(0)(3) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} + \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -10+1 & -17+8 \\ 2+4 & 4+2 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix} \rightarrow(2)$

From (1) and (2), we have

$A(B+C) = AB + AC$

(iv). $\boldsymbol{A(B-C) = AB - AC}$

Sol:

$L.H.S = A(BC)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1-2 & 2-1 \\ -3-1 & -5-3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$

$= \begin{bmatrix} (-1)(-1)+(3)(-4) & (-1)(1)+(3)(-8) \\ (2)(-1)+(0)(-4) & (2)(1)+(0)(-8) \end{bmatrix}$

$= \begin{bmatrix} 1-12 & -1-24 \\ -2+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \ldots(1)$

$R.H.S = AB - BC$

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) - \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$- \begin{bmatrix} (-1)(2)+(3)(1) & (-1)(1)+(3)(3) \\ (2)(2)+(0)(1) & (2)(1)+(0)(3) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} - \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$

$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -10-1 & -17-8 \\ 2-4 & 4-2 \end{bmatrix}$

$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \rightarrow(2)$

From (1) and (2), we have

$A(B-C) = AB - AC$

Q#6) For the matrices $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$, verify that

(i). $\boldsymbol{(AB)^t = B^tA^t}$

Sol: : $\boldsymbol{L.H.S = (AB)^t}$

First we find AB

$AB = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}$

$AB = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$

Taking transpose on both side

$(AB)^t = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}^t$

$(AB)^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \ldots(1)$

$: R.H.S = B^tA^t$

$= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} (1)(-1)+(-3)(3) & (1)(2)+(-3)(0) \\ (2)(-1)+(-5)(3) & (2)(2)+(-5)(0) \end{bmatrix}$

$= \begin{bmatrix} -1-9 & 2+0 \\ -2-15 & 4+0 \end{bmatrix}$

$B^tA^t = \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix} \rightarrow(2)$

From (1) and (2), we have

$(AB)^t = B^tA^t$

(ii). $\boldsymbol{(BC)^t = C^tB^t}$

Sol: $\boldsymbol{L.H.S = (BC)^t}$

First we find BC

$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

$= \begin{bmatrix} (1)(-2)+(2)(3) & (1)(6)+(2)(-9) \\ (-3)(-2)+(-5)(3) & (-3)(6)+(-5)(-9) \end{bmatrix}$

$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$

$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix} \rightarrow(2)$

Taking transpose on both side

$(AB)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$

$(AB)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (1)$

$\because R.H.S = C^tB^t$

$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$

$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

$= \begin{bmatrix} (-2)(1)+(3)(2) & (-2)(-3)+(3)(-5) \\ (6)(1)+(-9)(2) & (6)(-3)+(-9)(-5) \end{bmatrix}$

$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$

$C^tB^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (2)$

From (1) and (2), we have

$$(BC)^t = C^tB^t$$

Determinant of 2×2 matrix:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2×2 square matrix, the determinant of A is denoted by $|A|$ or detA

And given as

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= (a)(d) - (b)(c)$

$= ad - bc$

For example, $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2$

Singular and Non-singular matrices:

Singular matrix:

A square matrix A is called Singular matrix if its determinant is zero i.e. $|A| = 0$

For example, $A = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$= (3)(2) - (3)(2)$

$= 6 - 6 = 0$

Non-Singular matrix:

A square matrix A is called Non-Singular matrix if its determinant is not zero i.e. $|A| \neq 0$

For example, $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2 \neq 0$

Adjoint of Matrix A:

"Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries."

For example, $A = \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix}$

$AdjA = \begin{bmatrix} 0 & -4 \\ -2 & -1 \end{bmatrix}$

Exercise 1.5

Q#1) Find the determinant of the following matrices.

(i). $\boldsymbol{A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}}$

Sol:

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$

$= (-1)(0) - (1)(2)$

$= 0 - 2 = -2$

(ii). $\boldsymbol{B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}}$

Sol:

$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$

$= (1)(-2) - (3)(2)$

$= -2 - 6 = -8$

(iii). $\boldsymbol{C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}}$

Sol:

$|C| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$

$= (3)(2) - (3)(2)$

$= 6 - 6 = 0$

(iv). $\boldsymbol{D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}}$

Sol:

$|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

$= (3)(4) - (2)(1)$

$= 12 - 2 = 10$

Q#2)

Find which of the following matrices are singular or non-singular?

(i). $\boldsymbol{A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}}$

Sol:

$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$

$= (3)(4) - (6)(2)$

$= 12 - 12 = 0$

Hence, matrix A is singular matrix.

(ii). $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Sol:

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$

$= (4)(2) - (1)(3)$

$= 8 - 3 = 5$

Which is not zero and hence, matrix A is Non-singular matrix.

(iii). $\boldsymbol{C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}}$

Sol:

$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$
$= (7)(5) - (-9)(3)$
$= 35 + 27 = 62$
Which is not zero and hence, matrix A is Non-singular matrix.

(iv). $\boldsymbol{D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}}$

Sol:
$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$
$= (5)(4) - (-10)(-2)$
$= 20 - 20 = 0$
Hence, matrix A is singular matrix.

Q#3) Find the multiplicative inverse (if exists) of each:

(i). $\boldsymbol{A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}}$

Sol: First we find the determinant of A as
$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$
$= (-1)(0) - (3)(2)$
$= 0 - 6 = -6$
Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

As
$$A^{-1} = \frac{1}{|A|} AdjA$$
Putting values
$$A^{-1} = \frac{1}{-6} \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} \frac{0}{-6} & \frac{-3}{-6} \\ \frac{-2}{-6} & \frac{-1}{-6} \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii). $\boldsymbol{B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}}$

Sol: First we find the determinant of B as
$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$
$= (1)(-5) - (2)(-3)$
$= -5 + 6 = 1$
Which is not zero and hence, matrix B is Non-singular matrix and B^{-1} exist.

Now, $AdjB = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$

As
$$B^{-1} = \frac{1}{|B|} AdjB$$
Putting values
$$B^{-1} = \frac{1}{1} \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

(iii). $\boldsymbol{C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}}$

Sol: First we find the determinant of C as
$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$
$= (-2)(-9) - (3)(6)$
$= 18 - 18 = 0$
Which is zero and hence, matrix C is singular matrix and C^{-1} does not exist.

(iv). $\boldsymbol{D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}}$

Sol: First we find the determinant of D as
$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$
$= \left(\frac{1}{2}\right)(2) - \left(\frac{3}{4}\right)(1) = 1 - \frac{3}{4}$
$= \frac{4-3}{4} = \frac{1}{4}$
Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD = \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

As
$$D^{-1} = \frac{1}{|D|} AdjD$$
Putting values
$$D^{-1} = \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \frac{4}{1} \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$
$$D^{-1} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q#4) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then

(i). $\boldsymbol{A(AdjA) = (AdjA)A = (detA)I}$

Sol: First we find the determinant of A as
$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$
$= (1)(6) - (2)(4)$
$= 6 - 8 = -2$

Now, $AdjA = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

Let $A(AdjA) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$
$= \begin{bmatrix} (1)(6) + (2)(-4) & (1)(-2) + (2)(1) \\ (4)(6) + (6)(-4) & (4)(-2) + (6)(1) \end{bmatrix}$
$= \begin{bmatrix} 6 - 8 & -2 + 2 \\ 24 - 24 & -8 + 6 \end{bmatrix}$
$A(AdjA) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ ...(1)

And $(AdjA)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$
$= \begin{bmatrix} (6)(1) + (-2)(4) & (6)(2) + (-2)(6) \\ (-4)(1) + (1)(4) & (-4)(2) + (1)(6) \end{bmatrix}$

$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$

$(AdjA)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$...(2)

Also, $(detA)I = -2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$...(3)

From Eq(1), (2) and (3), we have

$A(AdjA) = (AdjA)A = (detA)I$

(ii). $BB^{-1} = B^{-1}B = I$

Sol: First we find the determinant of

$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$

$= (3)(-2) - (-1)(2)$

$= -6 + 2 = -4$

Now, $AdjB = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

As

$B^{-1} = \frac{1}{|B|}AdjB$

Putting values

$B^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

Let $BB^{-1} = \frac{1}{-4}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} (3)(-2)+(-1)(-2) & (3)(1)+(-1)(3) \\ (2)(-2)+(-2)(-2) & (2)(1)+(-2)(3) \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \rightarrow (1)$

Also $B^{-1}B = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} (-2)(3)+(1)(2) & (-2)(-1)+(1)(-2) \\ (-2)(3)+(3)(2) & (-2)(-1)+(3)(-2) \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$

$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \rightarrow (2)$

From (1) and (2), we have

$BB^{-1} = B^{-1}B = I.$

Q#5) Determine whether the given matrices are multiplicative inverse of each other or not.

(i). $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Sol:

$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} (3)(7)+(5)(-4) & (3)(-5)+(5)(3) \\ (4)(7)+(7)(-4) & (4)(-5)+(7)(3) \end{bmatrix}$

$= \begin{bmatrix} 21-20 & -15+15 \\ 28-28 & -20+21 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes, the given matrices are multiplicative inverse of each other.

(ii). $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Sol:

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1)(-3)+(2)(2) & (1)(2)+(2)(-1) \\ (2)(-3)+(3)(2) & (2)(2)+(3)(-1) \end{bmatrix}$

$= \begin{bmatrix} -3+4 & 2-2 \\ -6+6 & 4-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes, the given matrices are multiplicative inverse of each other.

Q#6) If $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$, then verify that

(i). $(AB)^{-1} = B^{-1}A^{-1}$

Sol: L.H.S=$(AB)^{-1}$

First we find

$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} (4)(-4)+(0)(1) & (4)(-2)+(0)(-1) \\ (-1)(-4)+(2)(1) & (-1)(-2)+(2)(-1) \end{bmatrix}$

$= \begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2-2 \end{bmatrix}$

$= \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$

Now, we find the its determinant

$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$

$= (-16)(0) - (-8)(6)$

$= 0 - (-48) = 48$

Which is not zero and hence, matrix AB is Non-singular matrix and $(AB)^{-1}$ exist.

Now, $AdjAB = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$

As

$(AB)^{-1} = \frac{1}{|AB|}AdjAB$

Putting values

$L.H.S = (AB)^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$...(1)

$R.H.S = B^{-1}A^{-1}$

First, we find B^{-1} and A^{-1}

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4)(2) - (0)(-1)$

$= 8 - 0 = 8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}AdjA$

Putting values

$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Also, $|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$

$= (-4)(-1) - (-2)(1)$

$= 4 + 2 = 6$
Which is not zero and hence, matrix B is Non-singular matrix and B^{-1} exist.

Now, $AdjB = \begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} AdjB$

Putting values

$B^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}$

$R.H.S = B^{-1}\, A^{-1}$

$= \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix} \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{8 \times 6}\begin{bmatrix} -1 & 2 \\ -`1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} (-1)(2) + (2)(1) & (-1)(0) + (2)(4) \\ (-1)(2) + (-4)(1) & (-1)(0) + (-4)(4) \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} -2 + 2 & 0 + 8 \\ -2 - 4 & 0 - 16 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$ …(2)

From (1) and (2), we have

$\boldsymbol{(AB)^{-1} = B^{-1}\, A^{-1}}$

(ii). $\boldsymbol{(DA)^{-1} = A^{-1}D^{-1}}$

Sol: L.H.S=$\boldsymbol{(DA)^{-1}}$

First we find

$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (3)(4) + (1)(-1) & (1)(0) + (1)(2) \\ (-2)(4) + (2)(-1) & (-2)(0) + (2)(2) \end{bmatrix}$

$= \begin{bmatrix} 12 - 1 & 0 + 2 \\ -8 - 2 & 0 - 4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$

Now, we find the its determinant

$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$

$= (11)(4) - (2)(-10)$

$= 44 + 20 = 64$

Which is not zero and hence, matrix DA is Non-singular matrix and $(DA)^{-1}$ exist.

Now, $AdjDA = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$

$(DA)^{-1} = \frac{1}{|DA|} AdjDA$

Putting values

$L.H.S = (DA)^{-1} = \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$ …(1)

$R.H.S = A^{-1}D^{-1}$

First, we find D^{-1} and A^{-1}

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4)(2) - (0)(-1)$

$= 8 - 0 = 8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} AdjA$

Putting values

$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Also, $|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$

$= (3)(2) - (1)(-2)$

$= 6 + 2 = 8$

Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$D^{-1} = \frac{1}{|D|} AdjD$

Putting values

$D^{-1} = \frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$R.H.S = A^{-1}D^{-1}$

$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{8 \times 8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} (2)(2) + (0)(-2) & (2)(-1) + (0)(3) \\ (1)(2) + (4)(-2) & (1)(-1) + (4)(3) \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4 + 0 & -2 + 0 \\ 2 + 8 & -1 + 12 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$ …(2)

From (1) and (2), we have

$(DA)^{-1} = A^{-1}\, D^{-1}$

Exercise 1.6

Q#1) Use matrices, to solve the following system of linear equations by:
(a). the matrix inverse method
(b). the Cramer's rule
(i). $2x - 2y = 4$; $3x + 2y = 6$

Sol: (a). the matrix inverse method
In matrix form

$$\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}, X = \begin{bmatrix}x\\y\end{bmatrix}$ and $B = \begin{bmatrix}4\\6\end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|}AdjA \rightarrow (2)$

$|A| = \begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$

$= (2)(2) - (-2)(3)$

$= 4 + 6 = 10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|}AdjA$

$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{10}\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix}(2)(4) + (2)(6)\\(-3)(4) + (2)(6)\end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix}8 + 12\\-12 + 12\end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix}20\\0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

$\Rightarrow \ x = 2, y = 0$

(b). the Cramer's rule
In matrix form

$$\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$$

Where $A = \begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}, A_x = \begin{bmatrix}4 & -2\\6 & 2\end{bmatrix}$ and

$A_y = \begin{bmatrix}2 & 4\\3 & 6\end{bmatrix}$

First of all we find $|A|, |A_x| and \ |A_y|$

$A = \begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$

$= (2)(2) - (-2)(3)$

$|A| = 4 + 6 = 10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix}4 & -2\\6 & 2\end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix}4 & -2\\6 & 2\end{vmatrix}$

$|A_x| = (4)(2) - (-2)(6)$

$|A_x| = 8 + 12 = 20$

Also,

$A_y = \begin{bmatrix}2 & 4\\3 & 6\end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}$

$= (2)(6) - (4)(3)$

$|A_y| = 12 - 12 = 0$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{20}{10} = 2$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{10} = 0$

Hence, $x = 2$ and $y = 0$

(ii). $2x + y = 3$; $6x + 5y = 1$

Sol: (a). the matrix inverse method
In matrix form

$$\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$...(1)

Where $A = \begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}, X = \begin{bmatrix}x\\y\end{bmatrix}$ and $B = \begin{bmatrix}3\\1\end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|}AdjA \rightarrow (2)$

$|A| = \begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$

$= (2)(5) - (1)(6)$

$= 10 - 6 = 4$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|}AdjA$

$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{4}\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix}(5)(3) + (-1)(1)\\(-6)(3) + (2)(1)\end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix}15 - 1\\-18 + 2\end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix}14\\-16\end{bmatrix} = \begin{bmatrix}\frac{14}{4}\\-\frac{16}{4}\end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$

$\Rightarrow \quad x = \frac{7}{2}, y = -4$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Where $A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$, $A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$ and

$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$

$= (2)(5) - (1)(6)$

$= 10 - 6 = 4$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$

$|A_x| = (3)(5) - (1)(1)$

$|A_x| = 15 - 1 = 14$

Also,

$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$

$= (2)(1) - (3)(6)$

$|A_y| = 2 - 18 = -16$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{14}{4} = \frac{7}{2}$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{-16}{4} = -4$

Hence, $x = \frac{7}{2}$ and $y = -4$

(iii). $4x + 2y = 8$; $3x - y = -1$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA$...(2)

$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} (-1)(8) + (-2)(-1) \\ (-3)(8) + (4)(-1) \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -8 + 2 \\ -24 - 4 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{7}{5} \end{bmatrix}$

$\Rightarrow \quad x = \frac{3}{5}, y = \frac{7}{5}$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$ and $A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$

$|A_x| = (8)(-1) - (2)(-1)$

$|A_x| = -8 + 2 = -6$

Also,

$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (8)(3)$

$|A_y| = -4 - 24 = -28$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-6}{-10} = \frac{3}{5}$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{-28}{-10} = \frac{7}{5}$

Hence, $x = \frac{3}{5}$ and $y = \frac{7}{5}$

(iv). $3x - 2y = -6$; $5x - 2y = -10$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA \rightarrow (2)$

$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$

$= (3)(-2) - (-2)(5)$

$= -6 + 10 = 4$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} (-2)(-6) + (2)(-10) \\ (-5)(-6) + (3)(-10) \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 12 - 20 \\ 30 - 30 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} -8 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$\Rightarrow x = -2, y = 0$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Where $A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$, $A_x = \begin{bmatrix} -6 & 2 \\ -10 & -2 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$

First of all we find $|A|, |A_x| and |A_y|$

$A = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$

$= (3)(-2) - (-2)(5)$

$= -6 + 10 = 4$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} -6 & 2 \\ -10 & -2 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} -6 & 2 \\ -10 & -2 \end{vmatrix}$

$|A_x| = (-6)(-2) - (2)(-10)$

$|A_x| = 12 - 20 = -8$

Also,

$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$

$= (3)(-10) - (-6)(5)$

$|A_y| = -30 + 30 = 0$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-8}{4} = -2$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{4} = 0$

Hence, $x = -2$ and $y = 0$

(iii). $3x - 2y = 4$; $-6x + 4y = 7$

Sol: The matrix inverse method

In matrix form

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA \rightarrow (2)$

$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$

$= (3)(4) - (-2)(-6)$

$= 12 - 12 = 0$

Which is zero and hence, matrix A is singular matrix and A^{-1} does not exist. No solution possible.

(vi). $4x + y = 9$; $-3x - y = -5$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA \rightarrow (2)$

$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1) - (-3)(1)$

$= -4 + 3 = -1$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

Putting values in eq (2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} (-1)(9) + (-1)(-5) \\ (3)(9) + (4)(-5) \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} -9 + 5 \\ 27 - 20 \end{bmatrix}$

$= \frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{-4}{-1} \\ \frac{7}{-1} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$

$\Rightarrow \quad x = 4, y = -7$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Where $A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$

$= (4)(-1) - (-3)(1)$

$= -4 + 3 = -1$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}$

$|A_x| = (9)(-1) - (1)(-5)$

$|A_x| = -9 + 5 = -4$

Also,

$|A_y| = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix}$

$= (4)(-5) - (9)(-3)$

$A_y = -20 + 27 = 7$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-4}{-1} = 4$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{7}{-1} = 7$

Hence, $x = 4$ and $y = 7$

(vii). $\mathbf{2x - 2y = 4\ ;\ \ -5x - 2y = -10}$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow(1)$

Where $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA \rightarrow(2)$

$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} (-2)(4) + (2)(-10) \\ (5)(4) + (2)(-10) \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -8 - 20 \\ 20 - 20 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{-14} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow \quad x = 2, y = 0$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} \mathbf{2} & \mathbf{-2} \\ \mathbf{-5} & \mathbf{-2} \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \mathbf{4} \\ \mathbf{-10} \end{bmatrix}$$

Where $A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$

$= (2)(-2) - (-2)(-5)$

$= -4 - 10 = -14$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 4 & -2 \\ -10 & -2 \end{vmatrix}$

$|A_x| = (4)(-2) - (-2)(-10)$

$|A_x| = -8 - 20 = -28$

Also,

$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix}$

$= (2)(-10) - (4)(-5)$

$|A_y| = -20 + 20 = 0$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-28}{-14} = 2$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{-14} = 0$

Hence, $x = 2$ and $y = 0$

(viii). $3x - 4y = 4$; $x + 2y = 8$

Sol: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|} AdjA \ldots(2)$

$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$

$= (3)(2) - (-4)(1)$

$= 6 + 4 = 10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

Putting values in eq.(2), we have

$A^{-1} = \frac{1}{|A|} AdjA$

$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} (2)(4) + (4)(8) \\ (-1)(4) + (3)(8) \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 8 + 32 \\ -4 + 24 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$\Rightarrow x = 4, y = 2$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Where $A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$ and

$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$

First of all, we find $|A|, |A_x| and |A_y|$

$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$

$= (3)(2) - (-4)(1)$

$= 6 + 4 = 10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$

$= (4)(2) - (-4)(8)$

$|A_x| = 8 + 32 = 40$

Also,

$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$

$= (3)(8) - (4)(1)$

$|A_y| = 24 - 4 = 20$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{40}{10} = 4$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{20}{10} = 2$

Hence, $x = 4$ and $y = 2$